EXAMPLE 1: basic optimization

We have agent, who has 120 units of production factor L (**E: like Endowment**)
and spend them to buy products X and Y. The budget constraint 120 = 1X + 2Y determines the condition PX/PY=1/2=MRS=MRT.

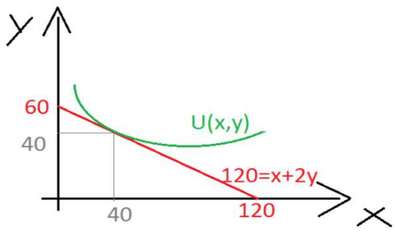
Thus the production function of good X - using (**I: like Input**) 100% of L (production factor) to make 100% of good X (**O: like output**)
The production function of good Y - using 200% of L to make 100% of good Y

Cobb-Douglas utility function $U(x,y)=X*Y^2$ or
$U(x,y)=\ln(x)+2\ln(y)$

=> consumer prefer 2-times more Y than X if Px=Py

$$MRS_{xy} = MRS_{1,1} = \frac{MU_x}{MU_y} = \frac{y}{2x} = \frac{1}{2*1} = \frac{1/2}{1}$$

$$MRS_{xy} = \frac{P_x}{P_y} = \frac{1}{2}$$ => Y=X because Px is cheaper



Budget constraint 120 = PX *X+PY *Y = PL*X + 2PL*Y = 3PL*X => solution is **X=40 Y=40**

Note: 1)The same solution we will obtain for $U(x,y)=X^{1/2}*Y$ or $U(x,y)=X^N*Y^{2N}$ (the difference will be only in the level of utility)
2)by default all functions in MPSGE are represented by Leontief structure, i.e. s:0
3)by default all variables in MPSGE are normalized to 1

		LOWER	LEVEL	UPPER	MARGINAL
----	VAR X	.	**40.000**	+INF	.
----	VAR Y	.	**40.000**	+INF	.
----	VAR PX	.	1.000	+INF	.
----	VAR PY	.	2.000	+INF	.
----	VAR PL	.	1.000	+INF	.
----	VAR RA	.	120.000	+INF	.

```
 $ONTEXT
$MODEL:DEMAND
$SECTORS:
        X        ! ACTIVITY LEVEL FOR X = DEMAND FOR GOOD X
        Y        ! ACTIVITY LEVEL FOR Y = DEMAND FOR GOOD Y
$COMMODITIES:
        PX       ! PRICE OF  X WHICH WILL EQUAL PL
        PY       ! PRICE OF  Y WHICH WILL EQUAL 2 PL
        PL       ! PRICE OF THE ARTIFICIAL FACTOR L
$CONSUMERS:
        RA       ! REPRESENTATIVE AGENT INCOME
$PROD:X
        O:PX     Q:1
        I:PL     Q:1
$PROD:Y
        O:PY     Q:1
        I:PL     Q:2

$DEMAND:RA       s:1
        E:PL     Q:120
        D:PX     Q:1       P:(1/2)
        D:PY     Q:1       P:1
$OFFTEXT
$SYSINCLUDE mpsgeset DEMAND
$INCLUDE DEMAND.GEN
SOLVE DEMAND USING MCP;
```

Conclusion: The relationship PX/PY in the budget constraint is determined by production technology (through MRT) and utility function (through MRS), but not through calibration point.

Supplement Material to EXAMPLE 1:

The original model assumes that Cobb-Douglas utility function can be represented in the model by the following elements:

- MRS(1,1)= ½ => MRS(x,y)=y/2x
- unit elasticity of substitution (s=1)

1) Prove that it is not possible to have MRS= X/Y for Cobb- Douglas function

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SOLUTION

The general form of Cobb-Douglas function is $\boldsymbol{U = aX^bY^c}$
where *a,b,c >0* are the coefficients.

Thus MRS for such function is

$$MRS = \frac{abX^{(b-1)}Y^c}{acXbY^{(c-1)}} = \frac{bY}{cX}$$

Special case when *b=c*:

$$MRS = \frac{Y}{X}$$

Conclusion: MRS=$\frac{MU_X}{MU_Y}$ for Cobb-Douglas function will never has the relationship X/Y

2) Prove that it is not possible to have MRS=y/x for Cobb-Douglas function when MRS (1,1)= ½

SOLUTION

If MRS=$\frac{Y}{X}$ => b=*c* => MRS (1,1)=1 => MRS (1,1)≠ ½
MRS (1,2)=2
MRS (3,1)=1/3

If MRS (1,1)=1/2 => MRS=$\frac{Y}{2X}$ => 2b=*c* => MRS (1,1) =1/2
MRS(1,2) = 1
MRS(3,1) = 1/6

Conclusion: MRS(1,1)=1/2 will never has the form of Y/X, but Y/(2X) because $\boldsymbol{U = aX^bY^{2b}}$ or $\boldsymbol{U = aX^{1/2c}Y^c}$. While for MRS(1,1)=1=Y/X we have $\boldsymbol{U = aX^bY^b}$ or $\boldsymbol{U = aX^cY^c}$

3) Prove that MRS (1,1)= ½ for Cobb-Douglas function means MRS=y/2x and this is the only solution.

SOLUTION

$$MRS_{1,1}=\frac{1}{2} \Rightarrow MRS=\frac{bY}{cX}=\frac{1}{2} \Rightarrow \frac{b*1}{c*1}=\frac{1}{2} \Rightarrow b=1 \ \& \ c=2 \Rightarrow MRS=\frac{Y}{2X} \Rightarrow U(x,y)=xy^2$$

MRS=1/2 for other Cobb-Douglass functions is achievable at other than (1,1) calibration points:

U(x,y)= xy^2 => MRS= y/2x => **MRS(1,1)= ½**
U(x,y)=xy => MRS=y/x => MRS(2,1)= ½
U(x,y) =x^2y => MRS=2y/x => MRS(½,1)=½
U(x,y) =xy^4 => MRS=y/4x => MRS(1,2) = ½

Conclusion: the calibration point (1,1) can obtain the value MRS=½ in the case of Cobb-Douglas function only when MRS=y/2x.

4) Modify the model in order to obtain PL≠1 for the original solution X=Y=40

SOLUTION

Several ways to achieve it.

Way 1: we can fix PY=1, i.e. PY becomes a numeraire

The relationship between prices of goods and inputs will not change:

- PX/PY=1/2
- PL/PY= 1/2

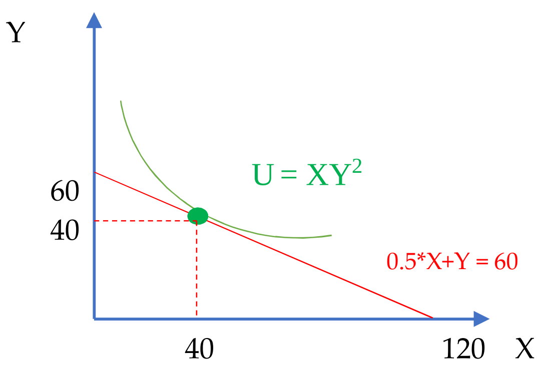
but the levels will change in comparison to the original case:

- PY = 2PL = 1=const => PL = 1/2
- PX = **PL = 0.5**

The optimum solution is determined by the price relationship (not the price levels) => the results for X and Y will not change:

MRSx,y = MUx/MUy = PX/PY =>
Y/2X = ½ => Y = X

PX*X+PY*Y = PL*120
PL*X+2PL*Y=PL*120
PL=1/2 =>
1/2*X+2*1/2*Y=1/2*120
1.5X=60
X=40 and Y=40
Conclusion: new PY => new PX,PL,RA



part of the MPSGE :

```
$OFFTEXT
$SYSINCLUDE mpsgeset DEMAND

PY.FX=1;

$INCLUDE DEMAND.GEN
SOLVE DEMAND USING MCP;
```

RESULTS :

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR X	.	**40.000**	+INF	.
---- VAR Y	.	**40.000**	+INF	.
---- VAR PX	.	0.500	+INF	.
---- VAR PY	1.000	1.000	1.000	EPS
---- VAR PL	.	0.500	+INF	.
---- VAR RA	.	60.000	+INF	.

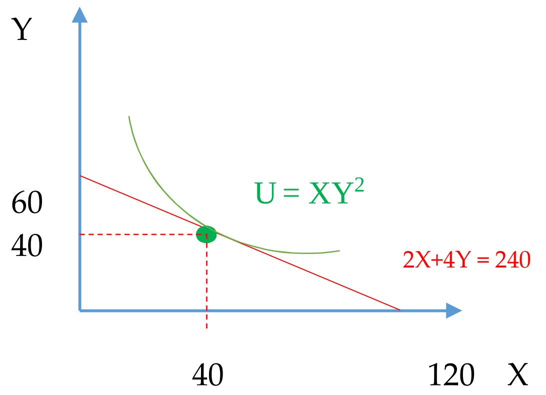
<u>Way 2:</u> we can fix PX=2, i.e. PX becomes a numeraire

The relationship between prices of goods and inputs will not change:

- PY/PX=2 => PY=4
- PL/PX= 1 => PL=2

PX*X+PY*Y = PL*120
PL*X+2PL*X =PL*120
2*X +2*2*X =2*120
6X=240
X=40 and Y=40

Conclusion: new benchmark price for one product => proportional rescale of all prices



part of the MPSGE :

```
$OFFTEXT
$SYSINCLUDE mpsgeset DEMAND

PX.FX=2;

$INCLUDE DEMAND.GEN
SOLVE DEMAND USING MCP;
```

RESULTS :

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR X	.	**40.000**	+INF	.
---- VAR Y	.	**40.000**	+INF	.
---- VAR PX	2.000	2.000	2.000	EPS
---- VAR PY	.	4.000	+INF	.
---- VAR PL	.	2.000	+INF	.
---- VAR RA	.	240.000	+INF	.

<u>Way 3</u>: we can change endowment from 120 to 100 and to fix it => income decreases and becomes a numeraire

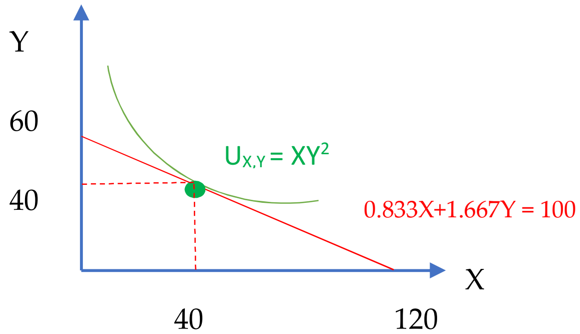
The relationship between prices and income will not be changed:

- PX/RA = 1/120=0.00833 => if RA=100, then PX = 0.833=PL
- PY/RA= 2/120=0.01667 => if RA=100, then PY = 1.667=2PL

Since the optimum solution is determined by the price relationship (not the price levels) => the results for X and Y will not be changed again, because MRS=const:

PX*X +PY*X = 100
PL*X +2PL*X =100
0.833*X +2*0.833*X=100
2.5X=100 ;
X=40 and Y=40

Conclusion: new endowment => proportional rescale of budget equation



part of the MPSGE :

```
$OFFTEXT
$SYSINCLUDE mpsgeset DEMAND

RA.FX=100;

$INCLUDE DEMAND.GEN
SOLVE DEMAND USING MCP;
```

RESULTS :

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR X	.	**40.000**	+INF	.
---- VAR Y	.	**40.000**	+INF	.
---- VAR PX	.	0.833	+INF	.
---- VAR PY	.	1.667	+INF	.
---- VAR PL	.	0.833	+INF	.
---- VAR RA	100.000	100.000	100.000	EPS

Main Conclusion: The only possibility to change the value of PL with no effect on the optimal allocation for goods X and Y is **(i)** to change a numeraire (i.e. to fix appropriately nominal variable) or **(ii)** to change price normalization (i.e. to set appropriately initial level of nominal variables). Any changes in production process or consumer preferences will not make the required effect in the model with a single production factor (i.e. no substitution effect is possible) and fixed labor supply (i.e. no leisure choice).

5) Modify the model to obtain results using multipliers instead of units

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SOLUTION

The original model is coded using multipliers (e.g. 40 units of X in results are represented by multiplier 1). If we want to obtain X=1 in results without modification of household income, we have to re-code the model using units instead of multipliers.

<u>**Case 1**</u> (the original):

- $U(x,y)=X*Y^2$ => we should ensure original MRS and original relationship between prices
- 120 = X + 2Y => 120*1 = 40 + 2*40 = 40 + 80
- X = min{Lx} and Y = min{½Ly} => 1 unit of L produce 1 unit of X, while 2 units of L produce 1 unit of Y no matter what the input prices are

  => TCx = PL*Lx =PL*X and TCy = PL*Ly = PL*2Y

  => MCx/MCy = ½
- 120 = Lx + Ly => 120 units of L should be splitted between X and Y => production possibility frontier 120 = X+ 2Y => 1 = X/120 + Y/60 => it can be created max 120 units of X or 60 units of Y

<u>**Case 2**</u> (results for all variables equal to 1):

- $U(x,y)=X*Y^2$ => we should ensure that PX=PY=PL
- 120 = X + Y => 120*1 = 40 + 80 due to consumer preferences $U(x,y)=X*Y^2$

  => we should modify production process in order to obtain equal prices
- X = min{Lx} and Y = min{Ly} => TCx = PL*Lx =PL*X and TCy = PL*Ly = PL*Y
- 120 = Lx + Ly => 120 = 40+80

<u>**Case 3**</u> (increased input demand):

- $U(x,y)=X*Y^2$
- 120 = X + 2Y => 120*1 = 40 + 2*40 = 40 + 80
- X = min{Lx} and Y = min{½Ly}
- 120 = Lx + Ly => 120 ≠ 40+160

  => since labor supply is fixed, labor demand should adjusts, i.e. 120=40+80

  => why the adjustment is done for Ly, but not for Lx? The reason is consumer preferences, where X=40 and Y=80 when Px=Py, but Px<Py, thus X=Y when 2Px=Py

  => why PL is not adjusted instead of (in addition to) Ly? If PL↑ => PY↑ and PX↑ => Y↓ and X↓ proportionally => ↓ demand on labor proportionally by 40%, i.e. 120=24+96=> X=24 and Y=96/2=48 when Px=PL=½Py, but consumers preferences are X=Y for such price relationship => such adjustment is not possible

part of the MPSGE code:

Case 1 (2PX=PY and X=Y)

```
$ONTEXT
$PROD:X
        O:PX     Q:40
        I:PL     Q:40
$PROD:Y
        O:PY     Q:40
        I:PL     Q:80
$DEMAND:RA       s:1
        E:PL     Q:120
        D:PX     Q:1       P:(1/2)
        D:PY     Q:1
$OFFTEXT
```

OR:

```
$ONTEXT
$PROD:X
        O:PX     Q:40
        I:PL     Q:40
$PROD:Y
        O:PY     Q:40
        I:PL     Q:80
$DEMAND:RA       s:1
        E:PL     Q:120
        D:PX     Q:40       P:(1/2)
        D:PY     Q:80
$OFFTEXT
```

RESULT:

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR X	.	1.000	+INF	.
---- VAR Y	.	1.000	+INF	.
---- VAR PX	.	1.000	+INF	.
---- VAR PY	.	2.000	+INF	.
---- VAR PL	.	1.000	+INF	.
---- VAR RA	.	120.000	+INF	.

Case 2 (PX=PY and 2X=Y)

```
$ONTEXT
$PROD:X
        O:PX     Q:40
        I:PL     Q:40
$PROD:Y
        O:PY     Q:80
        I:PL     Q:80
$DEMAND:RA       s:1
        E:PL     Q:120
        D:PX     Q:1       P:(1/2)
        D:PY     Q:1
$OFFTEXT
```

OR:

```
$ONTEXT
$PROD:X
        O:PX     Q:40
        I:PL     Q:40
$PROD:Y
        O:PY     Q:80
        I:PL     Q:80
$DEMAND:RA       s:1
        E:PL     Q:120
        D:PX     Q:40       P:(1/2)
        D:PY     Q:80
$OFFTEXT
```

RESULT:

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR X	.	1.000	+INF	.
---- VAR Y	.	1.000	+INF	.
---- VAR PX	.	1.000	+INF	.
---- VAR PY	.	1.000	+INF	.
---- VAR PL	.	1.000	+INF	.
---- VAR RA	.	120.000	+INF	.

Case 3 (2PX=PY and X=Y)

```
$ONTEXT
$PROD:X
        O:PX     Q:40
        I:PL     Q:40
$PROD:Y
        O:PY     Q:80
        I:PL     Q:160
$DEMAND:RA       s:1
        E:PL     Q:120
        D:PX     Q:1       P:(1/2)
        D:PY     Q:1
$OFFTEXT
```

OR:

```
$ONTEXT
$PROD:X
        O:PX     Q:40
        I:PL     Q:40
$PROD:Y
        O:PY     Q:80
        I:PL     Q:160
$DEMAND:RA       s:1
        E:PL     Q:120
        D:PX     Q:40       P:(1/2)
        D:PY     Q:80
$OFFTEXT
```

RESULT:

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR X	.	1.000	+INF	.
---- VAR Y	.	0.500	+INF	.
---- VAR PX	.	1.000	+INF	.
---- VAR PY	.	2.000	+INF	.
---- VAR PL	.	1.000	+INF	.
---- VAR RA	.	120.000	+INF	.

Results interpretation: no matter whether the model code is based on multipliers or results are based on multipliers, the proper interpretation of real term variables requires to multiply numbers from the model code with results. For example

- output by sector X in Case1 is 40*1 = 40
- input demand by X in Case1 is 40*1=40
- output by sector Y in Case1 is 40*1 = 40 => X=Y
- input demand by Y in Case1 is 80*1=80
- output by sector Y in Case2 is 80*1 = 80 => X≠Y even if results show the same multipliers
- input demand by Y in Case2 is 80*1=80
- output by sector Y in Case3 is 80*0.5 = 40 => X=Y even if results show different multipliers
- input demand by Y in Case3 is 160*0.5=80

Conclusions: (i) it is not possible to obtain multipliers in results for households income, but for all other variables; **(ii)** double supply in model code double this output in results (even if multipliers remain unchanged) and decreases its price (Case 2); **(iii)** double input demand in model code decreases output and increases its price if the input supply is fixed (Case 3 versus Case 2); **(iv)** double input demand and output in model code does not change results (even if multipliers change) if the input supply is fixed and production function is homothetic, i.e. homogenous of degree 1 (Case 3 versus Case 1); **(v)** it does not matter whether the model code or results are based on multipliers, if those numbers are properly interpreted.

6) Compare Cobb-Douglas versus Leontief utility function

Case 1: $U=X^{1/2}Y$ Case 3: $U=X^{1/3}Y^{2/3}$ Case 5: $U=\min\{X, ½Y\}$
Case 2: $U=XY^2$ Case 4: $U=\min\{2X,Y\}$ Case 6: $U=\min\{½X, ½Y\}$

SOLUTION

Cobb-Douglas utility function: $U(X,Y) = X^aY^b = (WM * pwm)^{a+b} = (UM)^{a+b}$

Cobb-Douglas unit expenditure function: $E(PX,PY) = (a+b)\left(\frac{PX}{a}\right)^{\frac{a}{a+b}}\left(\frac{PY}{b}\right)^{\frac{b}{a+b}}$

Cobb-Douglas total expenditure function: $ET(PX,PY,U) = E * U^{\frac{1}{a+b}} = M$

Cobb-Douglas demand (Marshallian) function: $X(PX,M) = \frac{a}{a+b}\frac{M}{PX}$ and $Y(PY,M) = \frac{b}{a+b}\frac{M}{PY}$

Cobb-Douglas indirect utility function: $V(PX,PY,M) = \left(\frac{a}{a+b}\frac{M}{PX}\right)^a\left(\frac{b}{a+b}\frac{M}{PY}\right)^b = U(X,Y)$

MPSGE Cobb-Douglas utility function: $UM(X,Y) = U^{\frac{1}{a+b}} = \frac{M}{E}$

MPSGE Cobb-Douglas price index for welfare: $PW = E = \frac{EM}{pwm}$

MPSGE Cobb-Douglas welfare output index: $pwm = (a)^{\frac{a}{a+b}}(b)^{\frac{b}{a+b}}$ or $pwm = a+b$

MPSGE Cobb-Douglas unit expenditure function: $EM(PX,PY) = (a+b)(PX)^{\frac{a}{a+b}}(PY)^{\frac{b}{a+b}} = E * pwm$

MPSGE Cobb-Douglas welfare (Hicksian) index: $WM = \frac{M}{EM} = \frac{UM}{pwm}$

Case 1	Case 2	Case 3
$U=X^{1/2}Y$ => MRS=Y/2X => MRS(1,1)=1/2 If Px/Py =1 => Y=2X If Px/Py =1/2 => Y=X $E = \left(1+\frac{1}{2}\right)\left(\frac{PX}{(1/2)}\right)^{\frac{1/2}{\frac{1}{2}+1}}\left(\frac{PY}{1}\right)^{\frac{1}{\frac{1}{2}+1}}$ => E(1,2)=3=PW pwm=0.8 EM(1,2)=2.4 $U(40,40)=253=(UM)^{(1+1/2)}$ **UM(40,40)=M/E =40**$=U^{1/(1+1/2)}$ WM=M/EM=50.4=UM/pwm	**$U=XY^2$** => MRS=Y/2X => MRS(1,1)=1/2 If Px/Py =1 => Y=2X If Px/Py =1/2 => Y=X $E = (2+1)\left(\frac{PX}{1}\right)^{\frac{1}{2+1}}\left(\frac{PY}{1}\right)^{\frac{1}{2+1}}$ => E(1,2)=3=PW pwm=1.6 EM(1,2)=4.8 $U(40,40)=64000=UM^{(1+2)}$ **UM(40,40)=M/E =40**$=U^{1/(1+2)}$ WM=M/EM=25.2=UM/pwm	**$U=X^{1/3}Y^{2/3}$** => MRS=Y/2X => MRS(1,1)=1/2 If Px/Py =1 => Y=2X If Px/Py =1/2 => Y=X $E = \left(\frac{1}{3}+\frac{2}{3}\right)\left(\frac{PX}{1/3}\right)^{\frac{1/3}{\frac{1}{3}+\frac{2}{3}}}\left(\frac{PY}{2/3}\right)^{\frac{2/3}{\frac{1}{3}+\frac{2}{3}}}$ => E(1,2)=3=PW pwm=0.5 EM(1,2)=1.6 $U(40,40)=40=UM^{(1/3+2/3)}$ **UM(40,40)=M/E =40**$=U^{1/(1/3+2/3)}$ WM=M/EM=75.6=UM/pwm

part of the MPSGE :

$DEMAND:RA s:1 E:PL Q:120 D:PX Q:1 P:(1/2) D:PY Q:1 $report: v:UM W:RA or **U=min{W}, where W=$X^{1/2}Y$** $PROD:WM s:1 O:PW Q:pwm I:PX Q:(1/2) I:PY Q:1 $DEMAND:RA E:PL Q:120 D:PW $report: v:UM W:RA	$DEMAND:RA s:1 E:PL Q:120 D:PX Q:1 D:PY Q:1 P:2 $report: v:UM W:RA or **U=min{W}, where W=XY^2** $PROD:WM s:1 O:PW Q:pwm I:PX Q:1 I:PY Q:2 $DEMAND:RA E:PL Q:120 D:PW $report: v:UM W:RA	$DEMAND:RA s:1 E:PL Q:120 D:PX Q:1 P(1/3) D:PY Q:1 P:(2/3) $report: v:UM W:RA or **U=min{W}, where W=$X^{1/3}Y^{2/3}$** $PROD:WM s:1 O:PW Q:pwm I:PX Q:(1/3) I:PY Q:(2/3) $DEMAND:RA E:PL Q:120 D:PW $report: v:UM W:RA

Leontief utility function: $U(X,Y) = \min\{aX, bY\}$

Leontief unit expenditure function: $E(PX,PY) = \frac{PX}{a} + \frac{PY}{b}$

Leontief total expenditure function: $ET(PX,PY,U) = E * U = M$

Leontief demand (Marshallian) function: $X(PX,M) = \frac{1}{a}\frac{M}{\frac{PX}{a}+\frac{PY}{b}}$ and $Y(PY,M) = \frac{1}{b}\frac{M}{\frac{PX}{a}+\frac{PY}{b}}$

Leontief indirect utility function: $V(PX,PY,M) = \frac{M}{\frac{PX}{a}+\frac{PY}{b}} = U(X,Y)$

MPSGE Leontief utility function: $UM(X,Y) = \min\left\{\frac{X}{1/a}, \frac{Y}{1/b}\right\} = \frac{M}{E} = U$

MPSGE Leontief price index for welfare: $PW = E$

MPSGE Leontief welfare output index: pwm= 1

MPSGE Leontief unit expenditure function: $EM(PX,PY) = E$

MPSGE Leontief welfare (Hicksian) index: $WM = UM = U$

<u>**Case 4**</u>	<u>**Case 5**</u>	<u>**Case 6**</u>
U=min{2X,Y} => 2X=Y no matter of Px/Py 120*PL=Px*X+Py*Y=PL*X+2PL*2X =5PL*X =>X=24, Y=48 $E = \frac{PX}{2} + \frac{PY}{1}$ => E(1,2)=2.5=PW **UM(24,48)=M/E =48**=U=WM	**U=min{X,½Y}** => X=½Y no matter of Px/Py 120*PL=Px*X+Py*Y=PL*½Y+2PL*Y =2.5PL*Y => Y=48, X=24 $E = \frac{PX}{1} + \frac{PY}{1/2}$ => E(1,2)=5=PW **UM(24,48)=M/E =24**=U=WM	**U=min{½X,½Y}** => X=Y no matter of Px/Py 120*PL=Px*X+Py*Y=PL*X+2PL*X =3PL*X => X=40, X=40 $E = \frac{PX}{1/2} + \frac{PY}{1/2}$ => E(1,2)=6=PW **UM(40,40)=M/E =20**=U=WM

part of the MPSGE :

\$DEMAND:RA E:PL Q:120 D:PX Q:(1/2) D:PY Q:1 \$report: v:UM W:RA or **U=min{W}, where W=min{2X,Y}** \$PROD:WM O:PW Q:1 I:PX Q:(1/2) I:PY Q:1 \$DEMAND:RA E:PL Q:120 D:PW \$report: v:UM W:RA	\$DEMAND:RA E:PL Q:120 D:PX Q:1 D:PY Q:2 \$report: v:UM W:RA or **U=min{W}, where W=min{X,½Y}** \$PROD:WM O:PW Q:1 I:PX Q:1 I:PY Q:2 \$DEMAND:RA E:PL Q:120 D:PW \$report: v:UM W:RA	\$DEMAND:RA E:PL Q:120 D:PX Q:2 D:PY Q:2 \$report: v:UM W:RA or **U=min{W}, where W=min{½X,½Y}** \$PROD:WM O:PW Q:1 I:PX Q:2 I:PY Q:2 \$DEMAND:RA E:PL Q:120 D:PW \$report: v:UM W:RA

Conclusions: (i) MPSGE rescales Cobb-Douglass utility and production functions in such a way that the same MRS but different share parameters give the same value of those functions. **(ii)** MPSGE rescales share parameters in Leontief utility and production function. **(iii)** MPSGE uses "P:" option just to define MRS. If MRS does not exist, like in Leontief case, then this option is ignored by MPSGE no matter what we will write there.

Exercise 1A:

```
a)Revise the program to use a different calibration point, where x=2 and y=1, where
MRS(2,1)=1/4.
```

Calibration point is the initial allocation.

The original example has $U(x,y)=X^*Y^2$ => MRS(1,1)=1/2 => MRS(2,1)=?

Use different calibration point, where x=2 and y=1 implies MRS(2,1)=y/2x=1/4, if we want to keep the same utility function. The initial allocation (2,1) means that consumer **has** 2 times more X than Y, while MRS=1/4 means that consumer **prefers** 4 times less X than Y when PX=PY. Taking it together we have MRS(1,1)=2/1*1/4=1/2. This means that when consumer **has** X=Y, he **prefers** to have 2 times less X than Y, i.e. $U(x,y)=X^*Y^2$.

Initial allocation: X/Y = 2/1		=>	Y = (1/2)X
	MRS(2,1)=1/4 = PX/PY	=>	PY= 4PX
	120 = PX*X+PY*Y	=>	120= PX*X + 4PX*(1/2)X = 3PX*X

=> X=40/PX

Final allocation: MRS(1,1) =Y/2X = 1/2		=>	Y = X
	PX/PY=1/2=MRT	=>	PY= 2PX
	120 = PX*X+PY*Y	=>	120= PX*X + 2PX*X = 3PX*X

=> X=40/PX

The solution does not change, because we modified the slopes of indifferent curve and initial allocation proportionally - we have 2 units of X for Price 1/4 (instead of 1 unit of X for Price 1/2)and 1 unit of Y for Price 1(as before).

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR X	.	**40.000**	+INF	.
---- VAR Y	.	**40.000**	+INF	.
---- VAR PX	.	1.000	+INF	.
---- VAR PY	.	2.000	+INF	.
---- VAR PL	.	1.000	+INF	.
---- VAR RA	.	120.000	+INF	.

```
$ONTEXT
$MODEL:DEMAND
……

$PROD:X
        O:PX    Q:1
        I:PL    Q:1
$PROD:Y
        O:PY    Q:1
        I:PL    Q:2

$DEMAND:RA      s:1
        E:PL    Q:120
        D:PX    Q:2     P:(1/4)
        D:PY    Q:1     P:1

$OFFTEXT
$SYSINCLUDE mpsgeset DEMAND
$INCLUDE DEMAND.GEN
SOLVE DEMAND USING MCP;
```

Conclusion: Optimal allocation is independent from calibration point (initial allocation), if MRS is adjusted appropriately (i.e. if utility function will not be changed).

Supplement Material to EXERCISE 1A:

What is the alternative solution for the Exercise 1A to obtain original results when calibration point is X=2 and Y=1, but MRS=1/2?

SOLUTION

Original model: max $\boldsymbol{U} = \boldsymbol{XY^2}$ s.t. X+2Y=120

- MRS(1,1) = ½ => consumer prefers Y over X if Px=Py
- MRS = Y/(2X)
- Px/Py=½ => X=Y because PX<PY

Exercise 1A: max $\boldsymbol{U} = \boldsymbol{XY^2}$ s.t. X+2Y=120

- MRS(2,1) = ¼ => MRS(1,1) = 2/1 * ¼= ½
- MRS = Y/(2X) => the same preferences as before
- Px/Py=½ => X=Y

Modified Exercise 1A:

- MRS(2,1) = ½ => MRS(1,1) = 2/1 * ½ =1 => $U=X^NY^N$
- MRS =Y/X => consumer prefers X and Y equally if Px=Py

In order to obtain the same solution as before when new utility function is applied, we have to change budget constraint:

- Px*40+Py*40=RA => Px/Py=1 if X=Y
- RA/Px=80 => **max U=XY** s.t. X+Y=80 or 1.5X+1.5Y=120

part of the MPSGE :

```
$PROD:X
        O:PX    Q:1
        I:PL    Q:1

$PROD:Y
        O:PY    Q:1
        I:PL    Q:1

$DEMAND:RA s:1
E:PL    Q:80
D:PX    Q:2     P:(1/2)
D:PY    Q:1     P:1
```

RESULTS:

	LOWER	LEVEL	UPPER
VAR X	.	40.000	+INF
VAR Y	.	40.000	+INF
VAR PX	.	1.000	+INF
VAR PY	.	1.000	+INF
VAR PL	.	1.000	+INF
VAR RA	.	80.000	+INF

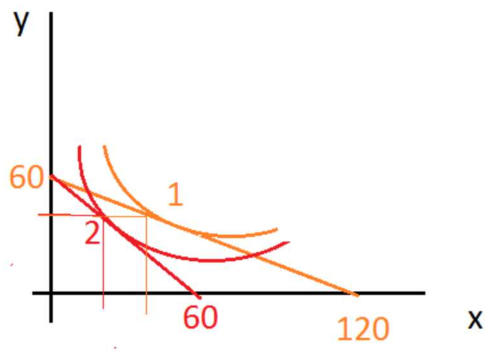
Conclusion: When utility function is modified, the only way to obtain the original results is to rescale budget constraint.

Exercise 1B:

Increase the price of x from 1 to 2 by changing the Q: coefficient for PL in sector X

We change the slope of the budget constraint, production function of good X becomes X(L)=2L, i.e. 2 units of labor to make 1 unit of good X. Thus MRS=y/2x=PX/PY=2/2. This means y=2x

New budget constrain: 120=2X+2Y (the red line)



$$\max U(x,y)= xy^2 \text{ s.t. } 2x+2y=120$$

$$L = xy^2 - \lambda(2x+2y-120)$$

$$\frac{dL}{dx} = y^2 - 2\lambda = 0 \Rightarrow y^2 = 2\lambda \rightarrow \frac{y^2}{2} = \lambda$$

$$\frac{dL}{dy} = 2xy - 2\lambda = 0 \Rightarrow 2xy = 2\lambda \Rightarrow \frac{2xy}{2} = \lambda$$

$$\Rightarrow \frac{y^2}{2} = \frac{2xy}{2} \Rightarrow 2y^2 = 4xy \Rightarrow 2y = 4x$$

$$\Rightarrow y = 2x \Rightarrow \mathbf{y = 40}$$

$$\frac{dL}{d\lambda} = 2x + 2y - 120 = 0 \Rightarrow \mathbf{x = 20}$$

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR X	.	**20.000**	+INF	.
---- VAR Y	.	**40.000**	+INF	.
---- VAR PX	.	2.000	+INF	.
---- VAR PY	.	2.000	+INF	.
---- VAR PL	.	1.000	+INF	.
---- VAR RA	.	120.000	+INF	2.7940E-8

```
$ONTEXT
$MODEL:DEMAND

……
$PROD:X
         O:PX     Q:1
         I:PL     Q:2
$PROD:Y
         O:PY     Q:1
         I:PL     Q:2

$DEMAND:RA       s:1
         E:PL     Q:120
         D:PX     Q:1      P:(1/2)
         D:PY     Q:1      P:1

$OFFTEXT
$SYSINCLUDE mpsgeset DEMAND

$INCLUDE DEMAND.GEN
SOLVE DEMAND USING MCP;
```

Conclusion: higher cost of production X implies higher PX, thus demand on X goes down. MRS=const because we did not shift along the indifferent curve, but parallel to another curve.

Supplement Material to EXERCISES 1A & 1B:

Compare nominal variables in the Exercises 1A and 1B when different numeraires are applied

Case 1: PX.FX=1 **Case 2:** PY.FX=1 **Case 3:** RA.fx=120

SOLUTION

Changes in numeraire implies that budget constraint PL*120=PX*X+PY*Y should be modified appropriately, while MRS(1,1)=Y/2X remains unchanged.

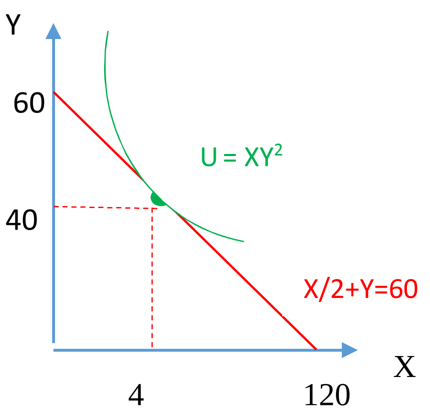
Exercise 1A:

Case 1

PX=1=const
If Py/Px = 2 => PY= 2
If PL/Px = 1 => PL= 1
RA= 120*PL = 120

MRSx,y = Y/2X = Px/Py = 1/2 => Y=X

120=X+2Y => Xmax=RA/PX=120
120=3X Ymax=RA/PY=60
X=40 and Y=40



Case 2

PY=1=const
If Px/Py = ½ => PX= ½
If PL/Py = ½ => PL= ½
RA= 120*PL = 60

60=½X+Y or 120= X+2Y
=> **the same results as in Case 1 for real variables, but not for nominal variables**

Case 3

RA=120=const => PL=1
PL=PX => Px=1
Py=2Px => PY=2

120= X+2Y
=> **the same results as in Case 1**

part of the MPSGE :

```
$OFFTEXT
$SYSINCLUDE mpsgeset DEMAND
PX.FX=1;
```

RESULT:

```
---- VAR X   .        40.000
---- VAR Y   .        40.000
---- VAR PX 1.000      1.000   1.000
---- VAR PY  .         2.000
---- VAR PL   .        1.000
---- VAR RA   .      120.000
```

```
$OFFTEXT
$SYSINCLUDE mpsgeset DEMAND
PY.FX=1;
```

RESULT:

```
---- VAR X   .        40.000
---- VAR Y   .        40.000
---- VAR PX  .         0.500
---- VAR PY 1.000      1.000   1.000
---- VAR PL   .        0.500
---- VAR RA   .       60.000
```

```
$OFFTEXT
$SYSINCLUDE mpsgeset DEMAND
RA.FX=120;
```

RESULT:

```
VAR X   .        40.000
VAR Y   .        40.000
VAR PX  .         1.000
VAR PY            2.000
VAR PL   .        1.000
VAR RA 120.000 120.000 120.000
```

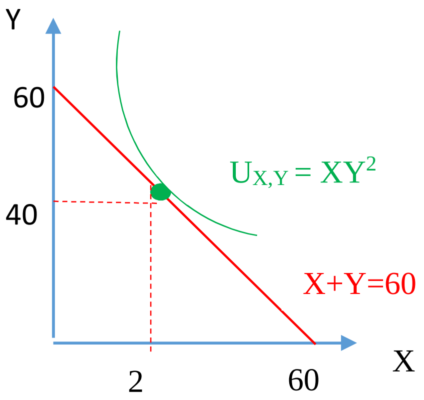
Exercise 1B:

Case 1

PX=1=const
If Py/Px = 1 => PY= 1
If PL/Px = ½ => PL= ½
RA= 120*PL = 60

MRSx,y = Y/2X = Px/Py = 1 => Y=2X

60=X+Y => Xmax=RA/PX=60
60=3X Ymax=RA/PY=60
X=20 and Y=40



Case 2

PY=1=const
If Px/Py = 1 => PX= 1
If PL/Py = ½ => PL= ½
RA= 120*PL = 60

=> **the same results as in Case 1**

Case 3

RA=120=const => PL=1
PX=PY=2PL=2

120= 2X+2Y
60 = X+Y
=> **the same results as in Case 1 for real variables, but not for nominal variables**

part of the MPSGE code:

```
$OFFTEXT
$SYSINCLUDE mpsgeset DEMAND
PX.FX=1;
```

RESULT:

```
---- VAR X   .          20.000
---- VAR Y   .          40.000
---- VAR PX 1.000        1.000   1.000
---- VAR PY .            1.000
---- VAR PL   .          0.500
---- VAR RA   .         60.000
```

```
$OFFTEXT
$SYSINCLUDE mpsgeset DEMAND
PY.FX=1;
```

RESULT:

```
---- VAR X   .          20.000
---- VAR Y   .          40.000
---- VAR PX .            1.000
---- VAR PY 1.000        1.000   1.000
---- VAR PL   .          0.500
---- VAR RA   .         60.000
```

```
$OFFTEXT
$SYSINCLUDE mpsgeset DEMAND
RA.fx=120;
```

RESULT:

```
VAR X   .          20.000
VAR Y   .          40.000
VAR PX  .           2.000
VAR PY              2.000
VAR PL    .         1.000
VAR RA 120.000 120.000 120.000
```

Conclusion: (i) PX.fx and Py.fx has identical consequences for nominal variables in the Exercise 1B because sector X and Y have identical technologies. As a consequence, both numeraires cause decrease in nominal income. **(ii)** PX.fx and RA.fx have identical consequences for nominal variables in the Exercise 1A because both numeraire assume the same level of nominal income. PY.fx generates lower nominal income due to labor intensive technology (PL is the price that measure income).

Exercise 1C:

Compute an equilibrium in which commodity y is defined as the numeraire

By default, MPSGE uses a numeraire for income of the richest household. Here we have only one household, thus the numeraire was RA.fx=120 (it is not displayed since it is a default setting)

By changing numeraire, the agent budget constraint changes: PL*120=PX*X+1*Y. Since MRS=y/2x =1/2, we have PL*120 = PL*X+2PL*Y, where 2PL=PY=1. Thus PL=1/2=PX => 1.5X=60 => X=40.

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR X	.	**40.000**	+INF	.
---- VAR Y	.	**40.000**	+INF	.
---- VAR PX	.	0.500	+INF	.
---- VAR PY	1.000	1.000	1.000	EPS
---- VAR PL	.	0.500	+INF	.
---- VAR RA	.	60.000	+INF	.

We set lower and upper level of PY on 1.00 => Other values in model must adapt to this numeraire definition.

```
$ONTEXT
$MODEL:DEMAND

$SECTORS:
        X       ! ACTIVITY LEVEL FOR X = DEMAND FOR GOOD X
        Y       ! ACTIVITY LEVEL FOR Y = DEMAND FOR GOOD Y

$COMMODITIES:
        PX      ! PRICE OF X WHICH WILL EQUAL PL
        PY      ! PRICE OF Y WHICH WILL EQUAL 2 PL
        PL      ! PRICE OF THE ARTIFICIAL FACTOR L

$CONSUMERS:
        RA      ! REPRESENTATIVE AGENT INCOME

$PROD:X
        O:PX    Q:1
        I:PL    Q:1
$PROD:Y
        O:PY    Q:1
        I:PL    Q:2

$DEMAND:RA      s:1
        E:PL    Q:120
        D:PX    Q:1     P:(1/2)
        D:PY    Q:1     P:1

$OFFTEXT
$SYSINCLUDE mpsgeset DEMAND
PY.FX = 1;

$INCLUDE DEMAND.GEN
SOLVE DEMAND USING MCP;
```

Conclusion: No matter numeraire (i.e. volumes become the same in the results), important is only relationship between prices (but not the price levels).

Supplement Material to EXERCISE 1C:

What will happen if two price variables will be fixed simultaneously?

Case 1: PY.FX=1
Case 2: PY.FX=2
Case 3: PY.FX=1 and RA.fx=120
Case 4: PX.FX=1 and RA.fx=120
Case 5: PY.FX=1 and PX.fx=1

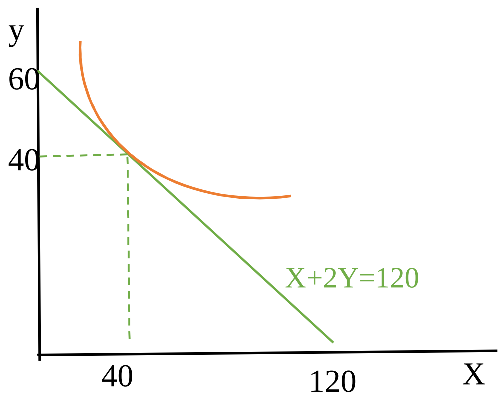
SOLUTION

Changes in numeraire implies that budget constraint PL*120=PX*X+PY*Y should be modified appropriately, while MRS(1,1)=1/2=PX/PY and X=Y remains unchanged.

Case original

RA=120=const => PL=1
PX=PL=1
PY=2PL=2

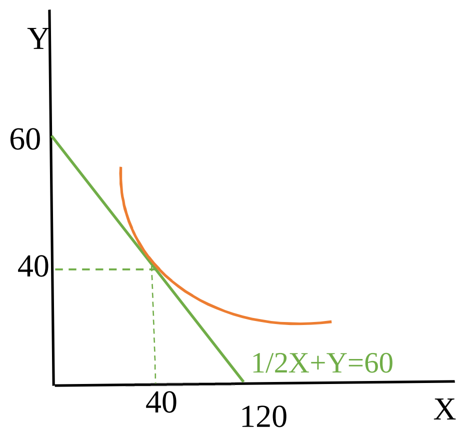
120= X+2*Y => Xmax = RA/PX=120
120= 3X Ymax = RA/PY=60
X=40=Y



Case 1

PY=1=const
If Px/Py = ½ => PX= ½=PL
RA= 120*PL = 60

60=1/2*X+Y => Xmax=RA/PX=120
60=3/2 *X Ymax=RA/PY=60
X=40=Y



Case 2

PY=2=const
If Px/Py = ½ => PX= 1=PL
RA= 120*PL = 120

120=X+2*Y

=> **the same results as in Case original**

part of the MPSGE code:

Case original

```
$OFFTEXT
$SYSINCLUDE mpsgeset DEMAND
RA.fx=120;
$INCLUDE DEMAND.GEN
SOLVE DEMAND USING MCP;
```

RESULT:

```
---- VAR X   .      40.000
---- VAR Y   .      40.000
--- VAR PX   .       1.000
---- VAR PY  .       2.000
---- VAR PL  .       1.000
---- VAR RA  120.000 120.000 120.000
```

The same results would be obtained for PX=1=const or PL=1=const

Case 1

```
$OFFTEXT
$SYSINCLUDE mpsgeset DEMAND
PY.fx=1;
$INCLUDE DEMAND.GEN
SOLVE DEMAND USING MCP;
```

RESULT:

```
---- VAR X  .       40.000
---- VAR Y  .       40.000
---- VAR PX .        0.500
---- VAR PY  1.000  1.000 1.000
---- VAR PL  .       0.500
---- VAR RA  .      60.000
```

The same results would be obtained for PX=0.5=const or PL=0.5=const

Case 2

```
$OFFTEXT
$SYSINCLUDE mpsgeset DEMAND
PY.fx=2;
$INCLUDE DEMAND.GEN
SOLVE DEMAND USING MCP;
```

RESULT:

```
---- VAR X  .      40.000
---- VAR Y  .      40.000
---- VAR PX .       1.000
---- VAR PY 2.000 2.000 2.000
---- VAR PL .       1.000
---- VAR RA  .    120.000
```

Case 3

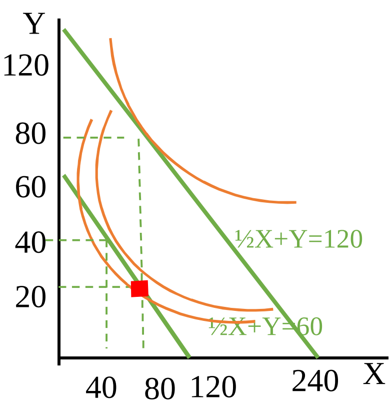
RA=120=const => PL=1=PX
PY=1 =const => PL= ½=PX
⇓
both conditions cannot be met simultaneously
⇓
numerical marginal values in the MPSGE results show bias.

Case 3a:
RA=120 =>PL=PX=1, PY=2
max $U=XY^2$ s.t. 120=X+2Y => **the same results as in Case original**

Case 3b:
PY=1, PL=PX=½ => RA=120/2=60
=> 120–60=60 units too many
max $U=XY^2$ s.t. 60 = ½*X+1*Y => **the same results as in Case 1**

Case 3c:
RA=120, PL=PY=PX=1
max $U=XY^2$ s.t. 120=X+Y
=> **the same results as in Case 5**

Case 3d (displayed by MPSGE):
RA=120, PL=1, PX=½, PY=1
1) max $U=XY^2$ s.t. 120=½X+Y
=> X=80=Y, but this is not feasible due to L=Lx+Ly=X+2Y
120 ≠ 80+2*80=240
⇓
not enough labor
Lx=80 and Ly=120-X=2Y => Y=(120-80)/2=20 => 20-80= -60 units too little
or
Lx=0 and Ly=120 => X=0, Y=60 (this is another possible solution)
or
Lx=120 and Ly=0 => X=120, Y=0 (this is another possible solution)



Case 4

RA=120=const => PL=1
PX=1=const
PY=2PL=2

120=X+2Y

=> **the same results as in Case original**

Similar effect will be obtained for:

- PY.FX=2 and RA.fx=120
- PL.FX=1 and RA.fx=120

but not for

- PX.FX=1, PY.FX=2, RA.fx=120
- PX.FX=1, PL.FX=1, RA.fx=120
- PL.FX=1, PY.FX=2, RA.fx=120

The reason that MPSGE is not able to find a solution when two prices are fixed is due to MCP approach:
1) equations that determines market clearing conditions are dropped automatically => no equations show a relationship between price and quantity
2) this is not an optimization process (no variable is maximized or minimized), but equilibrium => utility function is not defined at all, it is just a reporting variable in MCP programming

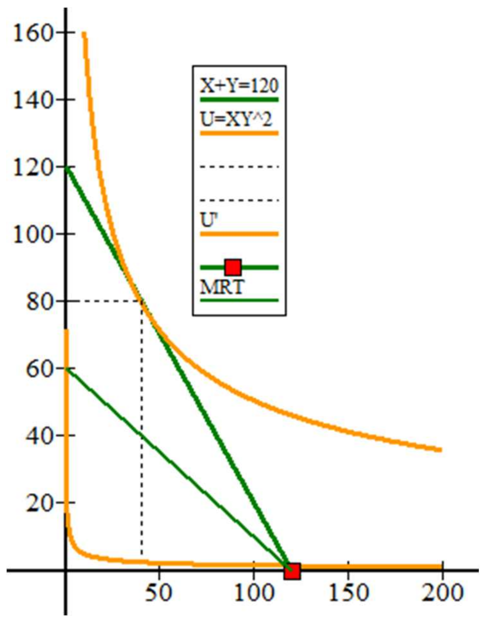
Algebraic version of MPSGE model for $U=XY^2$:
PL=PX => MC=MR for X
2PL=PY => MC=MR for Y
X=RA*1/3 / PX => S=D for X
Y=RA*2/3 / PY => S=D for Y
120=X+2Y => S=D for L
RA=120*PL => income
If we fix PX and PY, then MCP solver will not know the relationship between Px and X, Py and Y => it will not find the optimum. If we define utility function as a part of the model U=X*Y**2
(even if we do not maximise it in MCP), then the solver will find an optimal solution as an equilibrium.
Why, if variable U is not linked to any other equation and it is not a goal function?

Case 5

PY=1=const => PL= ½
PX =1=const => PL=1
⇓
both conditions cannot be met simultaneously
⇓
numerical marginal values in the MPSGE results show bias.

Case 5a:
PY=PX=1, PL= ½ => RA=120*PL=60
max $U=XY^2$ s.t. 60=X+Y
=>MRS=Y/2X = PX/PY = 1 => Y=2X
=> X=20, Y=40
but this is not feasible due to labor market L=Lx+Ly=X+2Y
120 ≠ 20+2*40=100 =>20 units too little

Case 5b (displayed by MPSGE):
RA=120, PY=PX=PL=1
max $U=XY^2$ s.t. 120=X+Y
=> X=40, Y=80
but this is not feasible due to labor market L=Lx+Ly=X+2Y
120 ≠ 40+2*80=200 =>80 units too many
⇓
not enough labor
Lx=0 and Ly=120=2Y => Y=60, X=0
or
Ly=0 and Lx=120=X => X=120, Y=0
=> too many units of X (120-40=80)



part of the MPSGE code:

Case 3 (infeasible)

```
$OFFTEXT
$SYSINCLUDE mpsgeset DEMAND
RA.fx=120;
PY.FX=1;
$INCLUDE DEMAND.GEN
SOLVE DEMAND USING MCP;
```

RESULT:

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR X	.	80.000	+INF	.
---- VAR Y	.	20.000	+INF	.
---- VAR PX	.	0.500	+INF	.
---- VAR PY	1.000	1.000	1.000	-60.000
---- VAR PL	.	0.500	+INF	.
---- VAR RA	120.000	120.000	120.000	60.000

Case 4

```
$OFFTEXT
$SYSINCLUDE mpsgeset DEMAND
RA.FX=120;
PX.FX=1;
$INCLUDE DEMAND.GEN
SOLVE DEMAND USING MCP;
```

RESULT:

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR X	.	40.000	+INF	.
---- VAR Y	.	40.000	+INF	.
---- VAR PX	1.000	1.000	1.000	EPS
---- VAR PY	.	2.000	+INF	.
---- VAR PL	.	1.000	+INF	.
---- VAR RA	120.000	120.000	120.000	EPS

Case 5 (infeasible)

```
$OFFTEXT
$SYSINCLUDE mpsgeset DEMAND
PY.FX=1;
PX.FX=1;
$INCLUDE DEMAND.GEN
SOLVE DEMAND USING MCP;
```

RESULT:

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR X	.	120.000	+INF	.
---- VAR Y	.	.	+INF	1.000
---- VAR PX	1.000	1.000	1.000	80.000
---- VAR PY	1.000	1.000	1.000	-80.000
---- VAR PL	.	1.000	+INF	.
---- VAR RA	.	120.000	+INF	.

Conclusions: (i) numeraire does NOT change the results of non-price variables and relative prices. **Income** is a price variable, so it changes with numeraire. **(ii)** if we have two fixed prices => one of them is a numeraire and another one is just a fixed variable => **non-price variables** and **relative prices** changes. **(iii)** when MPSGE displays marginal values and the level is not zero, this means that the solution is not optimal. Positive MARGINAL value informs about either too much value for real variable (displayed in the line for the complementary nominal variable) or too little value for nominal variable (displayed in the line for the complementary real variable) in the LEVEL of displayed results. **(iv)** the good practice is to run the same model with at least two alternative solvers in order to ensure global optimum and stability of results.